

# Static replications with traffic light options

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## Abstract

It is well known that any sufficiently regular one-dimensional payoff function has an explicit static hedge by bonds, forward contracts and lots of vanilla options. We show that the natural extension of the corresponding representation leads to a static hedge based on the same instruments along with traffic light options, which have recently been introduced in the market. One big advantage of these replication strategies is the easy structure of the hedge. Hence, traffic light options are particularly powerful building blocks for more complicated bivariate options. While it is well known that the second strike derivative of non-discounted prices of vanilla options are related to the risk-neutral density of the underlying asset price in the corresponding absolutely continuous settings, similar statements hold for traffic light options in sufficiently regular bivariate settings.

*Keywords:* correlation options, implied density, static replication, traffic light options

AMS Classifications: 60E05; 91G20

## 1 Introduction

Particularly in Europe, there has been a liquid market in structured products during recent years. At present, the majority of the trades still occur over the counter, but more and more trades are also organised at exchanges, especially at the quite new European exchange for structured products, Scoach. Quite often, structured products depend on more than one underlying asset. The increasing importance of quite complicated financial derivatives written on two or more underlying asset prices is also reflected in the fast growing literature about multivariate valuation technics, see e.g. the very recent article by Eberlein et al. [EGP10] and the literature cited therein. Also quite recently, several London-based investment banks have independently developed so-called traffic light options. In their purest form, traffic light options are the product of a standard equity put option and an interest rate floorlet. These products have been developed to suit the needs of Danish life and pension companies, see e.g. the paper by Jørgensen [Jør07]. More generally, traffic light options

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are European options with payoffs of the form

$$g_{k_1, k_2}(S_{T_1}, S_{T_2}) = \begin{cases} (k_1 - S_{T_1})_+(k_2 - S_{T_2})_+, \\ (k_1 - S_{T_1})_+(S_{T_2} - k_2)_+, \\ (S_{T_1} - k_1)_+(k_2 - S_{T_2})_+, \\ (S_{T_1} - k_1)_+(S_{T_2} - k_2)_+, \end{cases} \quad (1.1)$$

where  $S_{T_i}$ ,  $i = 1, 2$ , stand for two asset prices at maturity  $T > 0$ ,  $k_1, k_2 \geq 0$  are fixed strike prices, and  $(a)_+ = \max(a, 0)$  for  $a \in \mathbb{R}$ . Later, we will also use the notation  $\mathbb{R}_+ = [0, \infty)$ . In view of their increasing popularity in practice, also the theoretical interest in these products increased recently, see e.g. [Jør07, Kok09, Pou10]. Depending on the author, the same, or closely related options, are also called *correlation options* and have been analysed in relation to multivariate valuation techniques, see e.g. [BM00, DH02, EGP10, KLV10, Lee04, Zha01].

The theoretical popularity of these options is mainly based on the fact that they are relatively easy to price, while Bakshi and Madan [BM00] also remarked that the product of two calls on the price change factors is market completing in the sense of [Nac88, Ros76]. In this note, we emphasize two further, quite general but still explicit properties, being related to the latter one, which are analogues of very important and frequently used theoretical properties of vanilla options in the one-dimensional case. First, we show that in sufficiently regular absolutely continuous settings, the (implicit) joint risk-neutral density is directly related to the (non-discounted) prices of traffic light options, yielding a natural extension of a famous result due to Breeden and Litzenberger [BL78], namely that in sufficiently regular one-dimensional cases, the second strike derivatives of (non-discounted) prices of European vanilla options correspond to the risk-neutral probability density function. In view of the results in [CL10, HS90, Lip01], it turns out that traffic light options are more directly related to the (implicit) joint risk-neutral density than basket options. Furthermore, we show (in a purely analytical way, i.e. without having a probability space in the background) that traffic light options also have a great potential as building blocks for explicit static hedges. Especially in view of the observation that static hedges based on basket options seem to be harder to derive explicitly, c.f. e.g. [Bax98, HS90, Lip01] for certain families of functions, the easy structure of the resulting hedges seems to be a substantial advantage of traffic light options.

To sum up, traffic light options are quite easy to price, they directly reflect the joint risk-neutral distribution, and they are also efficient building blocks for static hedges, i.e. there are in fact important reasons to appreciate the market introduction of these products.

## 2 Traffic light options and the risk-neutral density

The observations concerning the probability density are quite obvious, but to our best knowledge, have not yet been reported in the existing literature.

**Proposition 2.1.** *Assume a risk-neutral absolutely continuous setting with continuous Lebesgue density denoted by  $q: \mathbb{R}_+^2 \rightarrow \mathbb{R}_+$ . Then for  $k_1, k_2 > 0$*

$$q(k_1, k_2) = \frac{\partial^4}{\partial k_1 \partial k_2 \partial k_1 \partial k_2} (\mathbf{E}[(k_1 - S_{T_1})_+(k_2 - S_{T_2})_+]).$$

*Proof.* Note that the expected payoffs in Proposition 2.1 are finite. Furthermore,

$$\begin{aligned}\mathbf{E}[(k_1 - S_{T1})_+(k_2 - S_{T2})_+] &= \int_{\mathbb{R}_+^2} (k_1 - x)_+(k_2 - y)_+ q(x, y) dx dy \\ &= \int_0^{k_2} \int_0^{k_1} (k_1 - x)(k_2 - y) q(x, y) dx dy.\end{aligned}$$

Differentiating yields the result.  $\square$

Note that in the above setting the (implicit risk-neutral) cumulative distribution function is obtained by

$$\mathbf{Q}(S_{T1} \leq k_1, S_{T2} \leq k_2) = \frac{\partial^2}{\partial k_1 \partial k_2} (\mathbf{E}[(k_1 - S_{T1})_+(k_2 - S_{T2})_+]).$$

*Remark 2.2* (Other traffic light and correlation options). Note that the integrals in the proof of Proposition 2.1 are only non-vanishing on bounded sets. In order to avoid problems being caused by integrating over non-bounded intervals in other cases, we impose some more regularity. More concretely, we assume that besides of the existence of a continuous joint density that the marginal distributions also exhibit continuous densities and that  $\int_{\mathbb{R}_+} x q(x, y) dx$  (respectively  $\int_{\mathbb{R}_+} y q(x, y) dy$ ) is continuous in  $y$  (respectively  $x$ ). Furthermore, we need  $\mathbf{E}[S_{T1} S_{T2}] < \infty$  (along with  $\mathbf{E}[S_{T1}]$ ,  $\mathbf{E}[S_{T2}] < \infty$ , imposing no extra restriction in our risk-neutral setting). Then, again by suitably writing down the expectations similarly as above and by differentiating, we obtain for  $k_1, k_2 > 0$

$$q(k_1, k_2) = \frac{\partial^4}{\partial k_1 \partial k_2 \partial k_1 \partial k_2} (\mathbf{E}[g_{k_1, k_2}(S_{T1}, S_{T2})]),$$

with  $g$  being either one of the functions given in (1.1).

As already mentioned, traffic light options are sometimes also called correlation options. However, more usually, correlation options represent options of the type

$$(S_{T1} - k_1)_+ \mathbb{I}_{S_{T2} > k_2}, \quad (k_1 - S_{T1})_+ \mathbb{I}_{S_{T2} < k_2}.$$

It almost goes without saying that the densities are also implicit in the prices of these options under the imposed assumptions. More concretely, differentiating the (non-discounted) prices (for arbitrary combinations of strikes) of these products two times with respect to  $k_1$  and once with respect to  $k_2$  yields (for the first product up to a minus sign) the joint risk-neutral probability density (in a corresponding absolutely continuous setting). Quite closely related to these observations is the well-known fact, see e.g. [Bru04, CLV04], that the prices of certain bivariate digital options also directly reflect the risk-neutral distribution. For example the (implicit) joint risk-neutral bivariate cumulative distribution function is given by  $\mathbf{Q}(S_{T1} \leq k_1, S_{T2} \leq k_2) = \mathbf{E}_{\mathbf{Q}}[\mathbb{I}_{S_{T1} \leq k_1} \mathbb{I}_{S_{T2} \leq k_2}]$ , i.e. by the non-discounted prices of a certain bivariate binary put (with arbitrary strike combinations).

Hence, theoretically, the non-discounted prices of a wide variety of traffic light options (or correlation options) easily yield the joint risk-neutral density. However, as in the one-dimensional setting, one has to keep in mind that since these expressions involve derivatives of (incomplete) market data, the canonical strategy leads to ill-posed problems and regularization will be needed.

### 3 Traffic light options in static hedges

In what follows, we use the convention  $\int_a^b f(x)dx = -\int_b^a f(x)dx$ . Furthermore, for  $A \subset \mathbb{R}^n$ , we say that  $f: A \rightarrow \mathbb{R}$  is differentiable of a certain order on  $A$  if  $f$  can be extended to a *differentiable function of the same order* on an open set  $U \supset A$ . We start with a short discussion of a well-known univariate result, due to Carr and Madan [CM98]. Assume, as in Bakshi and Madan [BM00], that a payoff function  $f: \mathbb{R}_+ \rightarrow \mathbb{R}$  is two times continuously differentiable (not necessarily integrable). As e.g. in [Lip01], by the fundamental theorem of calculus, by integration by parts, and by the formula  $xf'(x) = \int_a^x xf''(t)dt + xf'(a)$ , we have, for any  $a \in \mathbb{R}_+$

$$\begin{aligned} f(x) &= f(a) + \int_a^x f'(k)dk = f(a) + xf'(x) - af'(a) - \int_a^x kf''(k)dk \\ &= f(a) + \int_a^x xf''(k)dk + xf'(a) - af'(a) - \int_a^x kf''(k)dk \\ &= f(a) + f'(a)(x-a) + \int_a^x f''(k)(x-k)dk \\ &= f(a) + f'(a)(x-a) + \int_a^x f''(k)(x-k)_+dk - \int_a^x f''(k)(k-x)_+dk. \end{aligned}$$

We arrive at

$$f(x) = f(a) + f'(a)(x-a) + \int_a^\infty f''(k)(x-k)_+dk + \int_0^a f''(k)(k-x)_+dk, \quad (3.1)$$

$x \in \mathbb{R}_+$ , a well-known representation, see e.g. [BM00, CC97, CC02, CM98, HL09, Lip01]. The different original proof is presented in [CM98]. The economical interpretation of this representation is that if we let  $a$  be the current forward price, we have a static hedge with bonds, forwards, and lots of options (where the options are out of and at the money in a certain sense).

In the bivariate setting, we start by considering four times continuously differentiable (not necessarily integrable) payoff functions  $f: \mathbb{R}_+^2 \rightarrow \mathbb{R}$ . We introduce some efficient notation. By  $f_i$  we denote the partial derivative with respect to the  $i$ th component, by  $f_{ij}$  the second partial derivative calculated first with respect to the  $i$ th then with respect to the  $j$ th component, etc.

Similarly, as in the derivation of (3.1), we have for any fixed  $(a, b) \in \mathbb{R}_+^2$ ,  $(x, y) \in \mathbb{R}_+^2$

$$f(x, y) = -f(a, b) + f(x, b) + f(a, y) + \int_a^x \int_b^y f_{12}(k_1, k_2)dk_2dk_1.$$

By applying (3.1) for  $f(x, b)$  and  $f(a, y)$ , we arrive at

$$f(x, y) = I_1 + I_2 + I_3,$$

where

$$\begin{aligned} I_1 &= f(a, b) + f_1(a, b)(x-a) + f_2(a, b)(y-b) + \int_0^a f_{11}(k_1, b)(k_1-x)_+dk_1 \\ I_2 &= \int_a^\infty f_{11}(k_1, b)(x-k_1)_+dk_1 + \int_0^b f_{22}(a, k_2)(k_2-y)_+dk_2 + \int_b^\infty f_{22}(a, k_2)(y-k_2)_+dk_2 \end{aligned}$$

and

$$\begin{aligned}
I_3 &= \int_a^x \int_b^y f_{12}(k_1, k_2) dk_2 dk_1 \\
&= \int_a^x \left( y f_{12}(k_1, y) - b f_{12}(k_1, b) - \int_b^y k_2 f_{122}(k_1, k_2) dk_2 \right) dk_1 \\
&= \int_a^x \left( \int_b^y y f_{122}(k_1, k_2) dk_2 + y f_{12}(k_1, b) - b f_{12}(k_1, b) - \int_b^y k_2 f_{122}(k_1, k_2) dk_2 \right) dk_1 \\
&= \int_a^x \left( f_{12}(k_1, b)(y - b) + \int_b^y (y - k_2) f_{122}(k_1, k_2) dk_2 \right) dk_1 \\
&= \int_a^x f_{12}(k_1, b)(y - b) dk_1 + \int_a^x \int_b^y f_{122}(k_1, k_2)(y - k_2) dk_2 dk_1 = I_{31} + I_{32}.
\end{aligned}$$

The last two summands can be written as

$$\begin{aligned}
I_{31} &= (y - b) \int_a^x f_{12}(k_1, b) dk_1 \\
&= (y - b) \left( x f_{12}(x, b) - a f_{12}(a, b) - \int_a^x k_1 f_{121}(k_1, b) dk_1 \right) \\
&= (y - b) \left( \int_a^x x f_{121}(k_1, b) dk_1 + x f_{12}(a, b) - a f_{12}(a, b) - \int_a^x k_1 f_{121}(k_1, b) dk_1 \right) \\
&= (y - b) \left( f_{12}(a, b)(x - a) + \int_a^x f_{121}(k_1, b)(x - k_1) dk_1 \right) \\
&= (y - b) \left( f_{12}(a, b)(x - a) + \int_a^x f_{121}(k_1, b)(x - k_1)_+ dk_1 - \int_a^x f_{121}(k_1, b)(k_1 - x)_+ dk_1 \right) \\
&= f_{12}(a, b) \left( (x - a)_+(y - b)_+ - (x - a)_+(b - y)_+ - (a - x)_+(y - b)_+ + (a - x)_+(b - y)_+ \right) \\
&\quad + \int_a^\infty f_{121}(k_1, b) \left( (y - b)_+ - (b - y)_+ \right) (x - k_1)_+ dk_1 \\
&\quad + \int_0^a f_{121}(k_1, b) \left( (y - b)_+ - (b - y)_+ \right) (k_1 - x)_+ dk_1,
\end{aligned}$$

and, by applying Fubini's Theorem, along with the same arguments

$$\begin{aligned}
I_{32} &= \int_a^x \int_b^y f_{122}(k_1, k_2)(y - k_2) dk_2 dk_1 = \int_b^y (y - k_2) \int_a^x f_{122}(k_1, k_2) dk_1 dk_2 \\
&= \int_b^y (y - k_2) \left( f_{122}(a, k_2)(x - a) + \int_a^x f_{1221}(k_1, k_2)(x - k_1) dk_1 \right) dk_2 \\
&= (x - a) \int_b^y f_{122}(a, k_2)(y - k_2) dk_2 + \int_b^y \int_a^x f_{1221}(k_1, k_2)(y - k_2)(x - k_1) dk_1 dk_2 \\
&= \int_b^\infty f_{122}(a, k_2)((x - a)_+ - (a - x)_+)(y - k_2)_+ dk_2 \\
&\quad + \int_0^b f_{122}(a, k_2)((x - a)_+ - (a - x)_+)(k_2 - y)_+ dk_2 \\
&\quad + \int_b^y \int_a^x f_{1221}(k_1, k_2)(y - k_2)(x - k_1) dk_1 dk_2.
\end{aligned}$$

Finally, we rewrite the last summand as

$$\begin{aligned}
&\int_b^y \int_a^x f_{1221}(k_1, k_2)((y - k_2)_+ - (k_2 - y)_+)((x - k_1)_+ - (k_1 - x)_+) dk_1 dk_2 \\
&= \int_b^\infty \int_a^\infty f_{1221}(k_1, k_2)(x - k_1)_+(y - k_2)_+ dk_1 dk_2 \\
&\quad + \int_b^\infty \int_0^a f_{1221}(k_1, k_2)(k_1 - x)_+(y - k_2)_+ dk_1 dk_2 \\
&\quad + \int_0^b \int_a^\infty f_{1221}(k_1, k_2)(x - k_1)_+(k_2 - y)_+ dk_1 dk_2 \\
&\quad + \int_0^b \int_0^a f_{1221}(k_1, k_2)(k_1 - x)_+(k_2 - y)_+ dk_1 dk_2.
\end{aligned}$$

To sum up, we get for  $(x, y) \in \mathbb{R}_+^2$

$$\begin{aligned}
f(x, y) &= f(a, b) + f_1(a, b)(x - a) + f_2(a, b)(y - b) \\
&\quad + f_{12}(a, b)((x - a)_+(y - b)_+ + (a - x)_+(b - y)_+) \\
&\quad - f_{12}(a, b)((x - a)_+(b - y)_+ + (a - x)_+(y - b)_+) \\
&\quad + \int_0^a (f_{11}(k_1, b)(k_1 - x)_+ + f_{121}(k_1, b)((y - b)_+ - (b - y)_+)(k_1 - x)_+) dk_1 \\
&\quad + \int_a^\infty (f_{11}(k_1, b)(x - k_1)_+ + f_{121}(k_1, b)((y - b)_+ - (b - y)_+)(x - k_1)_+) dk_1 \\
&\quad + \int_0^b (f_{22}(a, k_2)(k_2 - y)_+ + f_{122}(a, k_2)((x - a)_+ - (a - x)_+)(k_2 - y)_+) dk_2 \\
&\quad + \int_b^\infty (f_{22}(a, k_2)(y - k_2)_+ + f_{122}(a, k_2)((x - a)_+ - (a - x)_+)(y - k_2)_+) dk_2 \\
&\quad + \int_0^b \left( \int_0^a f_{1221}(k_1, k_2)(k_1 - x)_+(k_2 - y)_+ dk_1 + \int_a^\infty f_{1221}(k_1, k_2)(x - k_1)_+(k_2 - y)_+ dk_1 \right) dk_2 \\
&\quad + \int_b^\infty \left( \int_0^a f_{1221}(k_1, k_2)(k_1 - x)_+(y - k_2)_+ dk_1 + \int_a^\infty f_{1221}(k_1, k_2)(x - k_1)_+(y - k_2)_+ dk_1 \right) dk_2.
\end{aligned} \tag{3.2}$$

The economical interpretation of this representation is that if we let  $a$  be the current forward price of the first and  $b$  the one of the second asset, we have a static hedge with bonds, forwards, and again lots of options, namely vanilla and traffic light options (being out of and at the money in a certain sense). However, the hedging formula simplifies considerably for  $a = b = 0$

$$\begin{aligned}
f(x, y) &= f(0, 0) + f_1(0, 0)x + f_2(0, 0)y + f_{12}(0, 0)xy \\
&+ \int_0^\infty f_{11}(k_1, 0)(x - k_1)_+ + f_{121}(k_1, 0)y(x - k_1)_+ dk_1 \\
&+ \int_0^\infty f_{22}(0, k_2)(y - k_2)_+ + f_{122}(0, k_2)x(y - k_2)_+ dk_2 \\
&+ \int_0^\infty \int_0^\infty f_{1221}(k_1, k_2)(x - k_1)_+(y - k_2)_+ dk_1 dk_2, \tag{3.3}
\end{aligned}$$

where the underlying possible asset prices  $x$  and  $y$  can also be interpreted as payoff of zero-strike options. This representation simplifies considerably for several concrete examples. E.g. if we would like to replicate some other well-known building blocks for certain families of functions or for approximations, we e.g. obtain for  $(x, y) \in \mathbb{R}_+^2$

$$\begin{aligned}
xy^2 &= \int_0^\infty 2x(y - k_2)_+ dk_2, \\
x^i y^j &= \int_0^\infty \int_0^\infty i(i-1)j(j-1)k_1^{i-2}k_2^{j-2}(x - k_1)_+(y - k_2)_+ dk_1 dk_2, \quad \text{where } i, j \geq 2,
\end{aligned}$$

or for  $h(x, y) = \frac{1}{2\pi} \exp(-\frac{1}{2}(x^2 + y^2))$ ,

$$\begin{aligned}
h(x, y) &= \frac{1}{2\pi} \left( 1 + \int_0^\infty (k_1^2 - 1)e^{-\frac{1}{2}k_1^2}(x - k_1)_+ dk_1 + \int_0^\infty (k_2^2 - 1)e^{-\frac{1}{2}k_2^2}(y - k_2)_+ dk_2 \right. \\
&\quad \left. + \int_0^\infty \int_0^\infty (1 + k_1^2 k_2^2 - k_1^2 - k_2^2)e^{-\frac{1}{2}(k_1^2 + k_2^2)}(x - k_1)_+(y - k_2)_+ dk_1 dk_2 \right).
\end{aligned}$$

It is stressed e.g. by Delbaen and Schachermayer [DS05] that a general analysis of financial markets should also consider situations where prices, at least for some instruments, can be negative. In view of that, it could be worth noticing that an only minimally modified version of the representation (3.2) also holds for four times continuously differentiable (not necessarily integrable) functions  $f: \mathbb{R}^2 \rightarrow \mathbb{R}$ . The modification is obtained by allowing  $a, b \in \mathbb{R}$  and by changing the integral limits from 0 to  $-\infty$  everywhere, where the limits are 0 in (3.2). The steps in the proof remain unchanged. However, note that the corresponding version of (3.3) and of the subsequent examples for general  $(x, y) \in \mathbb{R}^2$  remain slightly more complicated, since it is a priori not clear which terms in (3.2) vanish (this depends on the sign of  $x$  and  $y$ ). Furthermore, observe that the translates of  $h$  as function on  $\mathbb{R}^2$  are four times continuously differentiable and thus, can theoretically (i.e. the availability of the hedging instruments is assumed) be represented with the modified representation, while at the same time it is well known that the Fourier transform of  $h$  never vanishes. Hence, very similarly as in Bakshi and Madan [BM00], it follows from some extended (since here we are in the bivariate case) versions of Wiener's Approximation Theorem that based on [Rud62, Cor. 7.2.5d] (or even more directly based on [Rei68, Th. 4.1, Ch. 1]) the set of all finite linear combinations of translates of  $h$  are dense in  $L^1$ , or based on [Rud62, Th. 7.2.9] that translates of  $h$  also span  $L^2$ ,

i.e. the space of square integrable functions. Note that versions for real-valued integrable functions defined on  $\mathbb{R}^2$  can be obtained from the cited theorems as easy exercises, yielding the corresponding approximation results for real-valued  $L^1$ - respectively  $L^2$ -functions. Already these simple observations indicate the potential of the approach for approximating e.g. less regular functions. In the following, we give a similar result for integrable payoff functions depending only on two positive prices.

**Proposition 3.1.** *Integrable functions with representation (3.2) are dense in  $L^1(\mathbb{R}_+^2)$ .*

*Proof.* We extend  $f$  to a function  $F \in L^1(\mathbb{R}^2)$  by setting  $F(x) = f(x)$  for  $x \in \mathbb{R}_+^2$  and  $F(x) = 0$  otherwise. Further, we set  $h(x, y) = \frac{1}{2\pi} \exp(-\frac{1}{2}(x^2 + y^2))$ . As a consequence of Wiener's Approximation Theorem, we have that for arbitrary  $\varepsilon > 0$  there exist  $n \in \mathbb{N}$ ,  $\lambda_k$  and  $(a_k, b_k)_{k=1}^n$ , where  $\lambda_k, a_k, b_k \in \mathbb{R}$  such that for  $H: \mathbb{R}^2 \rightarrow \mathbb{R}$  defined as

$$H(x, y) = \sum_{k=1}^n \lambda_k h(x - a_k, y - b_k),$$

we obtain

$$\|H - F\|_{L^1(\mathbb{R}^2)} < \varepsilon.$$

We now define  $\tilde{h}_k: \mathbb{R}_+^2 \rightarrow \mathbb{R}$  by  $\tilde{h}_k(x, y) = h(x - a_k, y - b_k)$  and  $\tilde{h}(x, y) = \sum_{k=1}^n \lambda_k \tilde{h}_k(x, y)$ . Note that  $\tilde{h}$  is now only defined for  $(x, y) \in \mathbb{R}_+^2$ , and there it agrees with  $H$ . Hence, we can represent the functions  $\tilde{h}_k$  and also  $\tilde{h}$  the way we want. Furthermore,

$$\|f - \tilde{h}\|_{L^1(\mathbb{R}_+^2)} = \|F - H\|_{L^1(\mathbb{R}_+^2)} \leq \|F - H\|_{L^1(\mathbb{R}^2)} < \varepsilon.$$

□

Other interesting ideas concerning solving practical implementing problems can be found in other literature about the one dimensional case, see e.g. [CL09].

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